A general statement of the problem: In precision shooting the objective is to have the “smallest group size” for a given number of shots. The question is, what is the “smallest group size”?

One standard measure is simply the distance between the centers of the two shots farthest apart. While this is ok for match-day competition in determining the “winner,” it is not sufficient for the shooter who is trying to find the optimum combination of bullet size/weight, powder load, choice of primer, etc. that will make him or her competitive on match day. Intuitively, one wants the group that clusters the most bullet holes the closest to one another. One point of clarification - in this type of shooting, hitting the center of the target is not important - only the size of the resulting group. One uses the center of the target simply for sighting each shot. In fact, if one were to actually hit the center, one would destroy one’s aiming point and have trouble sighting subsequent shots. Hence, most groups are actually NOT centered on the aiming point.

Using some form of standard deviation of the distribution of shots is a natural tendency. Being as the target is a two dimensional object, and as one does not care if the shot is left, right, above or below center, but simply how far it is from the others, the distance from the group center to each shot is the proposed unit measure for statistical analysis. The proposed “measure of goodness” is

$$GroupDeviation = \sqrt{\frac{1}{(n-1)} \bullet \sum_{i=1}^{n} r_i^2}$$

shown in the equation above where “r_i” is the distance of the center of bullet hole “i” from the center of the group (“radius” in the lexicon of this discussion). This is an attempt to calculate the standard deviation of the radii of the shots in the group. A smaller standard deviation would intuitively be a good thing, and finding the combination of elements that results in a group with the smallest standard deviation sounds like a “winner” to me. However, since there are only positive distances from the center of the target (a radius “r” cannot be negative), the validity of this equation has been challenged as this cannot be a “normal” distribution.

The question then is, is it statistically valid to use the above equation to determine the standard deviation of the radii of shots in a group? If so, “good.” End of discussion. My argument in favor is based on the fact that in the calculation of standard deviation one squares the difference between each point and the mean anyway, so whether the point is left or right of middle is lost in the calculation anyway.

If not, a second question arises: who cares if it is not the standard deviation, if I call it by another name, is it a good measure of group size anyway?

If neither of these is statistically valid, a third alternative presents itself - the “x” and “y” positions of each shot ARE normally distributed around $\overline{x}$ and $\overline{y}$, respectively. Hence, one can find the standard deviation of the horizontal spread and the standard deviation of the vertical spread of the shots within a group.The next question is whether s_x and s_y can be combined or pooled to give any sort of meaningful measure of “goodness”. Pooling is calculated by $s_p = \sqrt{\frac{\sum_{i=1}^{k}((n_i-1) \bullet s_i^2)}{\sum_{i=1}^{k}(n_i-1)}}$, or in this case, $s_p = \sqrt{\frac{(n-1) \bullet s_x^2 + (n-1) \bullet s_y^2}{2 \bullet (n-1)}}$, since both x and y have the same number of shots. This, of course simplifies to $s_p = \sqrt{\frac{s_x^2 + s_y^2}{2}}$. From our friend Pythagorus, this looks like it could be a calculation of the standard deviation of radii.

Substituting in for s_x and s_y, we get $s_p = \sqrt{\frac{\left(\sqrt{\frac{1}{(n-1)} \bullet \sum_{i=1}^{n} (x_i - \bar{x})^2}\right)^2 + \left(\sqrt{\frac{1}{(n-1)} \bullet \sum_{i=1}^{n} (y_i - \bar{y})^2}\right)^2}{2}}$.

Simplifying, this becomes $s_p = \sqrt{\frac{1}{2(n-1)} \bullet \sum_{i=1}^{n} (x_i - \bar{x})^2 + (y_i - \bar{y})^2}$

And, if we define the center of the group as $(\bar{x}, \bar{y})$, then $(x_i - \bar{x})^2 + (y_i - \bar{y})^2 = r_i^2$. Hence, the adjusted proposal for the measure of group "goodness" is:

$$GroupDeviation = \sqrt{\frac{1}{2(n-1)} \bullet \sum_{i=1}^{n} r_i^2}$$